

number of degrees of freedom and variance estimates are shown in Table 19.4.

Some comment on the sums of squares in Table 19.4 is appropriate. The meaning of the row, column, and interaction sums of squares is obvious. These are concerned with variability due to the main effects and the interaction between the main effects. The subjects-within-groups sum of squares, S/R , is simply the variability among subjects for the first group, added to the variability among subjects for the second group, and so on for all levels of R . It may be viewed as the variability among subjects with the variability due to row treatment effects, as it were, removed. The $(C \times S)/R$ term is a column-by-subject interaction for the first group, added to the column-by-subject interaction for the second group, and so on for all levels of R .

The numbers of degrees of freedom associated with row, column, and $R \times C$ interaction sums of squares are $R - 1$, $C - 1$, and $(R - 1)(C - 1)$, respectively. The S/R term has associated with it $n - 1$ degrees of freedom for each group, and for R groups the number of degrees of freedom is $R(n - 1)$. The $(C \times S)/R$ term involves the summing of the $C \times S$ interaction over R groups or levels. The number of degrees of freedom associated with each level is $(n - 1)(C - 1)$; consequently the total number of degrees of freedom associated with this term is $R(n - 1)(C - 1)$.

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C_4	Means
X_{141}	$\bar{X}_{1.1}$
X_{142}	$\bar{X}_{1.2}$
X_{143}	$\bar{X}_{1.3}$
$\bar{X}_{14.}$	$\bar{X}_{1..}$
X_{244}	$\bar{X}_{2.4}$
X_{245}	$\bar{X}_{2.5}$
X_{246}	$\bar{X}_{2.6}$
$\bar{X}_{24.}$	$\bar{X}_{2..}$
$\bar{X}_{.4.}$	$\bar{X}_{...}$

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Table 19.4
Analysis of variance for two-factor experiments with repeated measurements on one factor

Source of variation	Sum of squares	df	Variance estimate
Between subjects	$C \sum^R \sum^n (\bar{X}_{r..} - \bar{X}_{...})^2$	$Rn - 1$	s_b^2
Rows	$nC \sum^R (\bar{X}_{r..} - \bar{X}_{...})^2$	$R - 1$	s_r^2
S/R	$C \sum^R \sum^n (\bar{X}_{r..} - \bar{X}_{r..})^2$	$R(n - 1)$	s_{sr}^2
Within subjects	$\sum^C \sum^R \sum^n (X_{rci} - \bar{X}_{r..})^2$	$Rn(C - 1)$	s_w^2
Columns	$nR \sum^C (\bar{X}_{.c.} - \bar{X}_{...})^2$	$C - 1$	s_c^2
$R \times C$	$n \sum^R \sum^C (\bar{X}_{rc.} - \bar{X}_{r..} - \bar{X}_{.c.} + \bar{X}_{...})^2$	$(R - 1)(C - 1)$	s_{rc}^2
$(C \times S)/R$	$\sum^C \sum^R \sum^n (X_{rci} - \bar{X}_{r..} - \bar{X}_{.c.} + \bar{X}_{...})^2$	$R(n - 1)(C - 1)$	s_{csr}^2
Total	$\sum^R \sum^C \sum^n (X_{rci} - \bar{X}_{...})^2$	$RCn - 1$	