19.7 TWO-FACTOR EXPERIMENTS WITH REPEATED MEASUREMENTS ON ONE FACTOR

A not uncommon type of experiment in psychological and educational research is a two-factor experiment with repeated measurements over one factor only. To illustrate, consider an experiment involving four different learning trials under two drug treatments. Two groups of n subjects each may be used. The first group may be tested on the four learning trials under the first drug treatment. The second group may be tested on the four learning trials under the second drug treatment. Designs of this type are sometimes called mixed designs, but this term should not be confused with mixed models, where the mixing is with respect to fixed and random factors rather than repeated- and nonrepeated-measurement factors.

The notation for such an experiment may be illustrated in the particular case where two experimental groups of three subjects are used with each subject measured under four experimental conditions. The data may be represented as follows:

		Subjects	C_1	C_2	C_3	C_4	Means
Group 1	R_1	1	X_{111}	X_{121}	X_{131}	X_{141}	$\bar{X}_{1.1}$
		2	X_{112}	X_{122}	X_{132}	X_{142}	$\bar{X}_{1.2}$
		3	X_{113}	X_{123}	X_{133}	X_{143}	$\bar{X}_{1.3}$
		Means	$\bar{X}_{11.}$	$\bar{X}_{12.}$	$\bar{X}_{13.}$	$\bar{X}_{14.}$	$\bar{X}_{1..}$
Group 2	R_2	4	X_{214}	X_{224}	X_{234}	X_{244}	$\bar{X}_{2.4}$
		5	X_{215}	X_{225}	X_{235}	X_{245}	$\bar{X}_{2.5}$
		6	X_{216}	X_{226}	X_{236}	X_{246}	$\bar{X}_{2.6}$
		Means	$\bar{X}_{21.}$	$\bar{X}_{22.}$	$\bar{X}_{23.}$	$\bar{X}_{24.}$	$\bar{X}_{2..}$
		Means	$\bar{X}_{.1.}$	$\bar{X}_{.2.}$	$\bar{X}_{.3.}$	$\bar{X}_{.4.}$	$\bar{X}_{...}$

Here triple subscripts are used. The first subscript identifies the row or group to which the subject belongs, the second subscript identifies the column or the level of the repeated measurement, the third subscript identifies the subject. For example, X_{214} is a measurement for the fourth subject in the second group at the first level of the repeated measurement.

For this type of experimental design the total sum of squares may be partitioned into two parts, a between-subjects and a within-subjects sum of squares. The between-subjects sum of squares can be further partitioned into two parts, a row sum of squares and a subjects-within-groups sum of squares. Denote this latter term by S/R. The within-subjects sum of squares can be further partitioned into three parts, a column sum of squares, a row-by-column interaction, and a third part which is a column-by-subject interaction pooled over groups or rows. Denote this latter term by $(S \times C)/R$. Thus, in effect, the total sum of squares is partitioned into five separate sums of squares. These sums of squares with the associated