are significant at the 5 percent level but fall short of significance at the 1 percent level.

19.5 TWO-FACTOR EXPERIMENTS WITH REPEATED MEASUREMENTS

In Sections 19.3 and 19.4 one-factor experiments with repeated measurements were considered. On occasion experiments are encountered that involve two factors with repeated measurements. Given R levels of one treatment and C levels of another, each subject may be tested under each of the RC treatments. If $R = 2$ and $C = 2$, the levels of R being R_1 and R_2 and of C being C_1 and C_2, there are four treatment combinations, R_1C_1, R_1C_2, R_2C_1, and R_2C_2. Each of N subjects might receive all the four treatments, the presentations being possibly, although not necessarily, arranged in random order for each subject.

Such data constitute an RCN block of numbers. Rows and columns are treatments, and layers are experimental subjects. These data are analyzed as in the triple-classification case with one observation in each cell. Use the computation formulas given in Section 17.8, writing $n = 1$. Seven sums of squares result: rows, columns, subjects, $R \times C$, $R \times S$, $C \times S$, and $R \times C \times S$. There is, of course, no within-cells sum of squares.

The model here is a mixed model with $n = 1$. Rows and columns will ordinarily be fixed variables. Layers, or subjects, is a random variable. For this model the expectation of the sums of squares and the degrees of freedom are as follows:

Mean square	Expectation of mean square	df
Rows, s_r^2	$\sigma_e^2 + C\sigma_{ac}^2 + NC\sigma_a^2$	$R - 1$
Columns, s_c^2	$\sigma_e^2 + R\sigma_{bc}^2 + NR\sigma_b^2$	$C - 1$
Subjects, s_s^2	$\sigma_e^2 + RC\sigma_c^2$	$N - 1$
$R \times C$, s_{rc}^2	$\sigma_e^2 + \sigma_{abc}^2 + N\sigma_{ab}^2$	$(R - 1)(C - 1)$
$R \times S$, s_{rs}^2	$\sigma_e^2 + C\sigma_{ac}^2$	$(R - 1)(N - 1)$
$C \times S$, s_{cs}^2	$\sigma_e^2 + R\sigma_{bc}^2$	$(C - 1)(N - 1)$
$R \times C \times S$, s_{rcs}^2	$\sigma_e^2 + \sigma_{abc}^2$	$(R - 1)(C - 1)(N - 1)$

Inspection of these expectations indicates that the appropriate error term for testing row effects is the $R \times S$ mean square, $F_r = s_r^2/s_{rs}^2$. The appropriate error term for testing column effects is the $C \times S$ mean square, $F_c = s_c^2/s_{cs}^2$. The appropriate error term for testing $R \times C$ interaction is the $R \times C \times S$ mean square, $F_{rc} = s_{rc}^2/s_{rcs}^2$. Unless the $R \times S$, $C \times S$, and $R \times C \times S$ interactions are assumed to be 0, which with most sets of data will not be the case, no unbiased test of differences between subjects, or $R \times S$ or $C \times S$ interactions, can be made. These are ordinarily not of interest.