

For the dental calculus data, the multivariate tests of the hypothesis that there is no TR effect (adjusted for the YEAR effect) are presented in Figure 1.32c.

Figure 1.32c

EFFECT .. TR					
MULTIVARIATE TESTS OF SIGNIFICANCE (S = 3, M = 0, N = 48)					
TEST NAME	VALUE	APPROX. F	HYPOTH. DF	ERROR DF	SIG. OF F
PILLAIS	.20122	1.79739	12.00	300.00	.048
HOTELLINGS	.22813	1.83769	12.00	290.00	.042
WILKS	.80733	1.82255	12.00	259.58	.045
ROYS	.14402				

The name of the test statistic is given under TEST NAME and its value listed under VALUE. For Pillai's criterion, Hotelling's trace, and Wilks lambda, approximate  $F$  statistics are given, with the degrees of freedom under HYPOTH. DF and ERROR DF and the  $p$ -values under SIG. OF F. A comparison (with references) of the powers of these four tests can be found in Morrison (1976).

- 3 Eigenvalues and canonical correlations. The nonzero eigenvalues of  $S_k S_k^{-1}$  and the corresponding canonical correlations for each effect in the model are given. For example, the results for the effect TR are shown in Figure 1.32d.

Figure 1.32d

EIGENVALUES AND CANONICAL CORRELATIONS				
ROOT NO.	EIGENVALUE	PCT.	CUM. PCT.	CANON. COR.
1	.16825	73.75366	73.75366	.37950
2	.05253	23.02709	96.78075	.22340
3	.00734	3.21925	100.00000	.08538

The canonical correlation coefficients  $\rho_i$  are calculated as  $\rho_i^2 = \lambda_i / (1 + \lambda_i)$ ; they are the canonical correlations between the response variables and the effect.  $\rho_i$  also measures the correlation between the  $i$ th canonical variate of the response variables and the tested effect (in certain linear combinations). The canonical correlations in this example can also be obtained by using the following dummy variables to represent the YEAR and TR effects.

$$X_i = 1 \text{ if YEAR} = 2 \\ 0 \text{ otherwise}$$

$$Y_1 = 1 \text{ if TR} = 2 \\ 0 \text{ otherwise}$$

$$Y_2 = 1 \text{ if TR} = 5 \\ 0 \text{ otherwise}$$

If  $X_i$  is already in the regression equation (since TR is adjusted for YEAR) and the within-cells SSCP matrix is used as the error matrix, then the  $\rho_i$ 's above are the canonical correlations between RCAN, RLI and RCI, and  $Y_1$ ,  $Y_2$ ,  $Y_3$ , and  $Y_4$ .

- 4 Dimension reduction analysis. Dimension reduction analysis, based on Wilks' lambda, is used to assess the dimensionality of a significant relationship between the response variables and the tested effect. The first test is based on all the eigenvalues and is equivalent to the overall Wilks' lambda test; the second test is performed on all the eigenvalues except the largest, and so on. Hence the value of Wilks' lambda for testing roots  $n_1$  to  $n_2$  is found by calculating the product from  $i = n_1$  to  $i = n_2$  of  $1/(1+\lambda_i)$ .

MANOVA also prints the approximate  $F$  statistic for each of these Wilks' lambda statistics. For the effect TR, the output in Figure 1.32e is obtained.

Figure 1.32e

DIMENSION REDUCTION ANALYSIS					
ROOTS	WILKS LAMBDA	F	HYPOTH. DF	ERROR DF	SIG. OF F
1 TO 3	.80733	1.82255	12.00	259.58	.045
2 TO 3	.94316	.97402	6.00	240.16	.443
3 TO 3	.99271	.36286	2.00	198.00	.696