Figure 1.32a

```
RUN NAME        DENTAL CALULUS DATA FROM FINN(1974) PAGE C-56
FILE NAME       DATA FOR ANTI-CALCULUS AGENT
VARIABLE LIST   YEAR,TR,RCAN,RLI,RCI,LCI,LLI,LCAN
INPUT FORMAT    FIXED(2F1.0,6F2.0)
N OF CASES      107
MISSING VALUES YEAR TO LCAN(BLANK)
MANOVA          RCAN,RLI,RCI BY YEAR(1,2),TR(1,5)/
READ INPUT DATA
11 2 2 1 2 2 1
11 0 0 0 2 1 0
11 0 0 4 4 0 0
11 2 2 2 3 2 2
 . . . . . .
 . . . . . .
 . . . . . .
23 0 1 3 4 3 0
23 1 0 1 0 1 0
23 0 1 0 0 0 0
23 0 1 6 4 1 0
FINISH
```

Since no DESIGN specifications are given in Figure 1.32a, a full factorial model is assumed. The standard output (without the PRINT subcommand) includes

1 General information about the design. This includes the number of observations, the number of levels of each effect, and the redundant effects (if any) in the model. This output is given in Figure 1.32b for the dental calculus data. (Three degrees of freedom are lost in the interaction effect because of empty cells.)

Figure 1.32b

```
     107 CASES ACCEPTED.
       0 CASES REJECTED BECAUSE OF OUT-OF-RANGE FACTOR VALUES.
       0 CASES REJECTED BECAUSE OF MISSING DATA.
       7 NON-EMPTY CELLS.

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CORRESPONDENCE BETWEEN EFFECTS AND COLUMNS OF BETWEEN-SUBJECTS DESIGN

 STARTING  ENDING
  COLUMN   COLUMN   EFFECT NAME

     1        1     CONSTANT
     2        2     YEAR
     3        6     TR
     7       10     YEAR BY TR

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REDUNDANCIES IN DESIGN MATRIX

   COLUMN     EFFECT

      8     YEAR BY TR
      9      (SAME)
     10      (SAME)
```

2 Multivariate tests of the significance of each effect in the model. The four test statistics previously mentioned are given. Each of these statistics is a function of the nonzero eigenvalues λ_i of the matrix $S_h S_e^{-1}$. The number of nonzero eigenvalues, s, is equal to the minimum of the number of dependent variables, q, and the degrees of freedom for the tested effect, n_h. The distributions of these statistics, under the null hypothesis, depend on q, n_h, and n_e (the error degrees of freedom).

Pillai's criterion. This test statistic, sum of $\lambda_i/(1+\lambda_i)$, can be approximated by an *F* variate (see Pillai, 1960). (The degrees of freedom are a function of q, n_h, and n_e.)

Hotelling's trace. This is the statistic T = sum of λ_i, which is equal to the trace of $S_h S_e^{-1}$. The critical points of the distribution of T have been tabulated by Pillai (1960) and depend on S = min(p,q), M = $(|n_h - q| - 1)/2$, and N = $(n_e - q - 1)/2$. (The values of S, M, and N for each effect are printed by MANOVA.) MANOVA also gives an approximate *F* statistic based on T, where the degrees of freedom depend on q, n_h and n_e.

Wilks' lambda. This test statistic, product of $1/(1+\lambda_i)$, can be transformed, using Rao's formula (Rao, 1973), into an approximate *F* statistic with degrees of freedom determined by q, n_h, and n_e.

Roy's largest root criterion. Upper percentage points of the distribution of this test statistic, $\lambda_1/(1+\lambda_1)$, where λ_1 is the largest eigenvalue of $S_h S_e^{-1}$, can be found in Heck (1960), Pillai (1967), and Morrison (1976). This distribution, like that of Hotelling's trace, depends on S, M, and N.