$(\beta\gamma)_{jk}$ is the error term for γ_j, $(\alpha\beta\gamma)_{ijk}$ is the error term for $(\alpha\gamma)_{ij}$, and if the number of observations per cell is greater than 1, then $(\alpha\beta)_{ik}$, $(\beta\gamma)_{jk}$ and $(\alpha\beta\gamma)_{ijk}$ can be tested against the within-cells error.

The following MANOVA specifications may be used to perform an analysis of variance of a 3 × 2 split-plot design:

```
MANOVA          Y BY BLOCK(1,3), A(1,3), B(1,2)/
                DESIGN=BLOCK, A VS 1, A BY BLOCK=1, B, A BY B/
```

In the above DESIGN specification, effect A is tested against the error 1 term which is the interaction of A and BLOCK. Effects B and AB are tested against the residual, since there is no within-cells error in this example.

This type of design can be extended by subdividing each subplot into sub-subplots, etc. The model for a split-split-plot design would be

$$Y_{ijkl} = \mu+\alpha_i+\beta_l+(\alpha\beta)_{il}+\gamma_j+(\alpha\gamma)_{ij}+\pi_{ijl}+\delta_k+(\alpha\delta)_{ik}+(\gamma\delta)_{jk}+(\alpha\gamma\delta)_{ijk}+\epsilon_{ijkl}$$

where δ_k is the effect for the sub-subplot factor, π_{ijl} = subplot residual = $(\gamma\beta)_{jl}$ + $(\alpha\gamma\beta)_{ijl}$, ϵ_{ijkl} is the residual, and $(\alpha\beta)_{il}$, π_{ijl}, and ϵ_{ijkl} are the appropriate error terms for plot, subplot, and sub-subplot factors, respectively.

An example of a split-split-plot design is taken from Hicks (1973, p. 223). The SPSS commands are given in Figure 1.27a.

Figure 1.27a

```
RUN NAME        SPLIT-SPLIT-PLOT DESIGN.
COMMENT         DATA ARE TAKEN FROM HICKS(1973) PAGE 223.
COMMENT         LAB : THREE DIFFERENT LABORATORIES--PLOT FACTOR.
COMMENT         TEM : THREE LEVELS OF TEMPERATURE--SUB-PLOT FACTOR.
COMMENT         MIX : THREE TYPES OF MIX--SUB-SUB-PLOT FACTOR.
COMMENT         FOUR REPLICATES (BLOCK).
VARIABLE LIST   BLOCK LAB TEM MIX RESP
N OF CASES      UNKNOWN
INPUT FORMAT    FIXED(4F1.0,1X,F4.1)
INPUT MEDIUM    CARD
MANOVA          RESP BY BLOCK(1,4),LAB,TEM,MIX(1,3)/
                DESIGN=BLOCK,LAB VS 1,LAB BY BLOCK=1,
                       TEM VS 2,LAB BY TEM VS 2,TEM BY BLOCK+
                       LAB BY TEM BY BLOCK=2,
                       MIX,LAB BY MIX,TEM BY MIX,LAB BY TEM BY MIX/
READ INPUT DATA
1111 18.6
1112 14.5
1113 21.1
1121  9.5
1122  7.8
1123 11.2
1131  5.4
1132  5.2
1133  6.3
1211 20.0
1212 18.4
1213 22.5
   .
   .
   .
4321  9.5
4322  9.0
4323 11.4
4331  4.8
4332  5.4
4333  5.8
END INPUT DATA
FINISH
```

As can be seen from the DESIGN specification, the interaction of LAB and BLOCK is the error term for the plot factor LAB, the interaction of TEM and BLOCK and the interaction of LAB, TEM, and BLOCK are pooled together as the error term for the subplot factors. The sub-subplot factors are to be tested against the residual. The analysis of variance from the output for this run is shown in Figure 1.27b.