If another restriction on the randomization is placed on a Latin square, we have a Graeco-Latin square. Table 1.23b exhibits a 4 × 4 Graeco-Latin square.

Table 1.23b

		Column			
		1	2	3	4
Row	1	A δ	B α	D β	C γ
	2	B γ	A β	C α	D δ
	3	C β	D γ	B δ	A α
	4	D α	C δ	A γ	B β

In this design the third restriction has levels α, β, γ, δ. Note that α, β, γ and δ not only each appear exactly once within each row and column, but they also appear exactly once with each level of treatments A, B, C, D. The Graeco-Latin square can be constructed by superimposing an orthogonal (same size) Latin square on the original Latin square. In other words, the third restriction factor along with column and row is also a 4 × 4 Latin square. It has treatments α, β, γ, δ and is orthogonal to the original Latin square with treatments A, B, C, and D. Here orthogonality means each letter in one Latin square appears exactly once in the same position as each letter of the other square.

The analysis of variance for a Graeco-Latin square is very similar to that for a Latin square. Let GREEK denote the third restriction factor on a 4 × 4 Graeco-Latin square. The MANOVA specifications would be

```
MANOVA          Y BY ROW(1,4), COL(1,4), GREEK(1,4), TRT(1,4)/
                DESIGN=ROW,COL,GREEK,TRT/
```

Note that a small Graeco-Latin square design may not be very practical, since very few degrees of freedom are left for the residual.

1.24 Factorial Designs

In a factorial design, the effects of several different factors are investigated simultaneously. Suppose we wish to study the effects of two factors on the yield of a chemical. The first factor is temperature at 100°F, 200°F, and 300°F. The other factor is pressure at 20 psi and 40 psi. This experiment is a two-factor factorial design with three levels for the first factor and two levels for the second. The treatments for this experiment are the 6 combinations of the levels of the factors. The model for the 3 × 2 factorial design is

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where α_i is the temperature effect, β_j is the pressure effect, and $(\alpha\beta)_{ij}$ is the temperature-pressure interaction.

A factorial experiment containing one observation per cell (treatment) constitutes one replicate of the design. The design may be replicated k times in two possible ways. If each observation has different experimental conditions for replications within cells (e.g., each replicate is a block), the design is crossed by another factor within k levels (i.e., block effect). If the experimental condition is the same for the replications within cells, the number of factors remains unchanged, and the variation within cells is attributed to the error.

The following example illustrates the use of MANOVA to perform the analysis of a 4 × 4 × 3 factorial in randomized blocks (two blocks) with a covariate. The data are taken from Cochran and Cox (1957, p. 176).

The model contains the main effects (NTREAT, LENPER, CURRENT), all two-way interactions (NTREAT BY LENPER,...,LENPER BY CURRENT), and the three-way interaction (NTREAT BY LENPER BY CURRENT). The SPSS commands for this analysis are shown in Figure 1.24a and the analysis of variance table in Figure 1.24b.