where

$$Z_{ijk} = \begin{cases} 1 & \text{if } i = 2 \text{ (level 2 of A is applied)} \\ 0 & \text{otherwise} \end{cases}$$

$$U_{ijk} = \begin{cases} 1 & \text{if } j = 2 \text{ (level 2 of B is applied)} \\ 0 & \text{otherwise} \end{cases}$$

If the interaction terms between the covariate and factor variables are added to the model, then

$$Y_{ijk} = \mu + \beta(X_{ijk} - \bar{X}) + \alpha_1 Z_{ijk} + \alpha_2 U_{ijk} + \alpha_3 Z_{ijk} U_{ijk} + (\alpha\beta)_1 (X_{ijk} - \bar{X}) Z_{ijk} + (\alpha\beta)_2 (X_{ijk} - \bar{X}) U_{ijk} + (\alpha\beta)_3 (X_{ijk} - \bar{X}) Z_{ijk} U_{ijk} + \epsilon_{ijk}$$

A test of H_0: $(\alpha\beta)_1 = (\alpha\beta)_2 = (\alpha\beta)_3 = 0$ is equivalent to testing the hypothesis that the regression slopes are the same for all cells. This test can be performed by specifying the following MANOVA commands:

```
MANOVA          Y X BY A(1,2),B(1,2)/
                ANALYSIS=Y/
                DESIGN=X,A,B,A BY B,
                  X BY A + X BY B + X BY A BY B/
```

The effects X BY A, X BY B, and X BY A BY B are lumped together to provide the test of the parallelism hypothesis. If the test is not significant, the usual analysis of covariance model can be used to perform the analysis.

If the assumption of the homogeneity of the slope is violated, one of the following three models might be used:

1 The model of different slopes for each level of factor A. This model can be justified by testing $(\alpha\beta)_2 = (\alpha\beta)_3 = 0$. The MANOVA specification for the test is

```
DESIGN=X,A,B,A BY B,
  X BY B + X BY A BY B/
```

If the test is not significant, the following DESIGN specifications can be used for the analysis of covariance of this model:

```
DESIGN=X WITHIN A, A, B, A BY B/
DESIGN=A, B, A BY B, X WITHIN A/
```

The X WITHIN A term represents the regression effects that are separately estimated within each level of A. The first DESIGN specification requests the main effects and interaction adjusted for the covariate effects. The second DESIGN specification gives the regression effect (last term) adjusted for A, B and AB.

2 The model of different slopes for each level of factor B. The appropriate test for this model is $(\alpha\beta)_1 = (\alpha\beta)_3 = 0$ and is obtained by specifying

```
DESIGN= X, A, B, A BY B, X BY A + X BY A BY B/
```

The analysis of covariance is obtained by using

```
DESIGN=X WITHIN B, A, B, A BY B/
DESIGN=A, B, A BY B, X WITHIN B/
```

3 The model of different slopes for each cell. The MANOVA specifications for this model are

```
DESIGN=X WITHIN A BY B, A, B, A BY B/
DESIGN=A, B, A BY B, X WITHIN A BY B/
```

The X WITHIN A BY B term represents the regression slopes, which are different for each cell.

The same procedure can be simply extended to multiple covariates. For a 2 × 2 design with covariates Z1 and Z2, the X term is replaced by CONTIN(Z1,Z2) throughout the DESIGN specification discussed above. The keyword CONTIN incorporates Z1 and Z2 into a single effect. Thus the following specfications may be used for the analysis of covariance for model 1 with covariates Z1 and Z2.

```
MANOVA          Y Z1 Z2 BY A(1,2) B(1,2)/
                ANALYSIS=Y/
                DESIGN=Z1, Z2, A, B, A BY B,
                   CONTIN(Z1,Z2) BY B + CONTIN(Z1,Z2)BY A BY B/
                DESIGN=CONTIN(Z1,Z2) WITHIN A, A, B, A BY B/
                DESIGN=A, B, A BY B, CONTIN(Z1,Z2) WITHIN A/
```

The first DESIGN specification is used to test the model, while the second and third models are for the analysis of covariance.