The second example is adapted from Cochran and Cox (1957, p. 46). The experiment was conducted to compare the effectiveness of four soil fumigants in keeping down the number of eelworms in the soil. The fumigants were CN, CS, CM, and CK. Each fumigant was tested both in a single and double dose. The control was used as another treatment. The nine treatments are denoted as C00 (control), CN1 (CN with single dose), CS1, CM1, CK1, CN2 (CN with double dose), CS2, CM2, and CK2. There were four replications for each dose of each fumigant and 16 replications of the control. The desired subdivisions of the treatment sum of squares are as follows:

1 If the effect of the fumigants is proportional to the dose, then both CN1 and CN2/2 are the estimate of the effect of CN per unit dose. The pooled estimate of this effect is (CN1+ 2(CN2))/5. The differences in the linear responses to the four fumigants can be measured by the following three contrasts:

```
(0  1 -1  0  0  2 -2  0  0)
(0  1  1 -2  0  2  2 -4  0)
(0  1  1  1 -3  2  2  2 -6)
```

2 The curvature of the treatment CN is measured by C00 − (2CN1) + CN2. The differences in curvature are compared by the quantities CN2 -2(CN1), (the C00 term cancelled out in the comparison) or by the following three contrasts:

```
(0  2 -2  0  0 -1  1  0  0)
(0  2  2 -4  0 -1 -1  2  0)
(0  2  2  2 -6 -1 -1 -1  3)
```

3 The sum of squares between levels (control: 0 level; treatments with single dose: level 1; treatments with double level: level 2) can be partitioned into a component due to the linearity between levels and one representing the curvature between levels. The former is given by the comparison of −1(level 0) +0(level 1) +1(level 2), or the contrast (−4 0 0 0 0 1 1 1 1). The curvature between levels is measured by 1(level 0) −2(level 1) +1(level 2), or the contrast (−4 −2 −2 −2 −2 1 1 1 1).

The above partitions can be summarized by the following MANOVA CONTRAST subcommand:

```
CONTRAST(TREATMNT)=SPECIAL(1  1  1  1  1  1  1  1  1
                           0  1 -1  0  0  2 -2  0  0
                           0  1  1 -2  0  2  2 -4  0
                           0  1  1  1 -3  2  2  2 -6
                           0  2 -2  0  0 -1  1  0  0
                           0  2  2 -4  0 -1 -1  2  0
                           0  2  2  2 -6 -1 -1 -1  3
                          -4  0  0  0  0  1  1  1  1
                           4 -2 -2 -2 -2  1  1  1  1)/
PARTITION(TREATMNT)=(3 3 1 1)/
DESIGN=BLOCK TREATMNT(1) TREATMNT(2)
       TREATMNT(3) TREATMNT(4)/
```

Note that TREATMNT(1), TREATMNT(2), TREATMNT(3), and TREATMNT(4) are the effects of the differences in linear response, in curvature, linear response between levels, and curvature between levels, respectively. Also, it can be verified that the effects are orthogonal.

1.17 Analysis of Covariance

SPSS MANOVA can perform an analysis of covariance in which interval-scaled independent variables (covariates) are used in conjunction with categorical variables (factors). Analysis of covariance is a technique that combines the features of analysis of variance and regression. A two-way analysis of covariance model with two covariates can be described as follows:

$$Y_{ijk} = \mu+\alpha_i+\gamma_j+(\alpha\gamma)_{ij}+\beta_1(X_{1ijk}-\bar{X}_1)+\beta_2(X_{2ijk}-\bar{X}_2)+\epsilon_{ijk}$$

where Y_{ijk} is the dependent variable, α_i, γ_j are the main effects, and $(\alpha\gamma)_{ij}$ is the interaction effect. X_1, X_2 are the covariates, and $\bar{X}_1$, $\bar{X}_2$ are the means for the two covariates.

In the covariance model, Y has a (multiple) linear regression (see Section 1.38) on X_1 and X_2 with regression coefficients β_1 and β_2. The regression procedure is used to remove the variation in the dependent variable due to covariates.

From the standpoint of the analysis of variance model, the covariate model is essentially an analysis of variance model on the corrected scores or

$$Y_{ijk}-\beta_1(X_{1ijk}-\bar{X}_1)-\beta_2(X_{2ijk}-\bar{X}_2) = \mu+\alpha_i+\gamma_j+(\alpha\gamma)_{ij}+\epsilon_{ijk}$$

which is the analysis of variance model for Y adjusted for the two covariates.

The following illustrative example is taken from Snedecor and Cochran (1967, p. 422). The model is a one-way analysis of covariance with one covariate. The experiment was conducted to evaluate the effect of three drugs on the treatment of leprosy. For each patient, six sites were selected. The variate X, based on laboratory tests, is a score representing the abundance of leprosy