1.16 Orthogonal Contrasts for Unequal Numbers of Replicates

For balanced designs, two treatment contrasts are orthogonal if the cross products of the contrast coefficients sum to zero. When treatments have unequal numbers of replicates, for contrasts to be orthogonal the weighted sum of cross products, where the weights are the reciprocals of the numbers of replicates, must be zero. For example, suppose the numbers of replicates for five treatments are 4, 2, 1, 5, and 1 respectively; then contrasts (4,2,-6, 0, 0) and (4, 2, 1, 5, -12) are orthogonal, since $4 \times 4/4 + 2 \times 2/2 + (-6)(1)/1 = 0$.

Figure 1.16a illustrates the use of the orthogonal contrasts in a one-way unbalanced design in which the numbers of observations for treatments are 4, 4, 1, and 1, respectively. Note that specification of the PARTITION command without degrees of freedom results in single-degree-of-freedom partitions.

Figure 1.16a

```
RUN NAME         ORTHOGONAL CONTRASTS FOR UNBALANCED DESIGN.
VARIABLE LIST    TREATMNT,Y
N OF CASES       10
INPUT MEDIUM     CARD
INPUT FORMAT     FIXED(F1.0,F2.0)
MANOVA           Y BY TREATMNT(1,4)/
                 PRINT=DESIGN(BIAS)/
                 CONTRAST(TREATMNT)=SPECIAL(1 1 1 1
                                            1 -1 0 0
                                            4 4 -8 0
                                            4 4 1 -9)/
                 PARTITION(TREATMNT)/
                 DESIGN=TREATMNT(1),TREATMNT(2),TREATMNT(3)/
READ INPUT DATA
1 8
1 7
2 8
2 9
310
1 6
1 7
2 8
2 6
4 9
FINISH
```

In this example, TREATMNT(1) defines a comparison between treatments 1 and 2; TREATMNT(2) is the contrast between treatment 3 and the combination of treatments 1 and 2; and TREATMNT(3) can be used to test the hypothesis that the average of the first three treatment effects is equal to the last treatment effect. All pairs of contrasts are orthogonal since $(1)(4)/4 + (-1)(4)/4 = 0$, $(1)(4)/4 + (-1)(4)/4 = 0$, and $(4)(4)/4 + (4)(4)/4 + (-8)(1)/1 = 0$. The F tests are therefore independent. The bias matrix and the ANOVA table corresponding to Figure 1.16a are given in Figure 1.16b.

Figure 1.16b

BIAS COEFFICIENTS FOR SEQUENTIAL ORDERING

EFFECT	EFFECT 1	2	3	4
1	10.00000	0.0	.00434	.00278
2	0.0	2.00000	0.0	0.0
3	0.0	0.0	.01389	0.0
4	0.0	0.0	0.0	.01111

TESTS OF SIGNIFICANCE FOR Y USING SEQUENTIAL SUMS OF SQUARES

SOURCE OF VARIATION	SUM OF SQUARES	DF	MEAN SQUARE	F	SIG. OF F
WITHIN CELLS	6.75000	6	1.12500		
CONSTANT	608.40000	1	608.40000	540.80000	0.0
TREATMNT(1)	1.12500	1	1.12500	1.00000	.356
TREATMNT(2)	6.12500	1	6.12500	5.44444	.058
TREATMNT(3)	1.60000	1	1.60000	1.42222	.278