The elements in the upper triangle of the decomposition matrix are used to obtain the sum of squares for each effect in the model. Consider a 2 × 3 factorial design, where T is the upper triangle of the decomposition matrix.

$$T = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} \\ 0 & t_{22} & t_{23} & t_{24} & t_{25} & t_{26} \\ 0 & 0 & t_{33} & t_{34} & t_{35} & t_{36} \\ 0 & 0 & 0 & t_{44} & t_{45} & t_{46} \\ 0 & 0 & 0 & 0 & t_{55} & t_{56} \\ 0 & 0 & 0 & 0 & 0 & t_{66} \end{pmatrix}$$

The first row of T represents the CONSTANT effect, the second row represents the effect of A, the third and fourth rows are the effects of B, and the last two rows are the effects of AB. If $\mathbf{h}' = (h_1\ h_2\ h_3\ h_4\ h_5\ h_6)$ is the least-squares éstimate of the contrasts of effects, then the sequential sums of squares for the effects are as shown in Table 1.13.

Table 1.13

Source	Sum of Squares
CONSTANT	$(t_{11}h_1+t_{12}h_2+t_{13}h_3+t_{14}h_4+t_{15}h_5+t_{16}h_6)^2$
A	$(t_{22}h_2+t_{23}h_3+t_{24}h_4+t_{25}h_5+t_{26}h_6)^2$
B adjusted A	$(t_{33}h_3+t_{34}h_4+t_{35}h_5+t_{36}h_6)^2+(t_{44}h_4+t_{45}h_5+t_{46}h_6)^2$
AB adjusted A,B	$(t_{55}h_5+t_{56}h_6)^2+(t_{66}h_6)^2$

If the DESIGN specification for this example is

```
DESIGN=A,B,A BY B/
```

then the bias matrix is a 4 × 4 upper triangular matrix, since the order of the bias matrix is the number of effects in the model (in this case, CONSTANT, A, B, and A BY B). The (i,j)th element of this matrix is obtained by summing the squared elements of the T matrix, which are in the rows of effect i and the columns of effect j. The bias matrix for this example is

$$\begin{pmatrix} t_{11}^2 & t_{12}^2 & t_{13}^2 + t_{14}^2 & t_{15}^2 + t_{16}^2 \\ 0 & t_{22}^2 & t_{23}^2 + t_{24}^2 & t_{25}^2 + t_{26}^2 \\ 0 & 0 & t_{33}^2 + t_{34}^2 + t_{44}^2 & t_{35}^2 + t_{36}^2 + t_{45}^2 + t_{46}^2 \\ 0 & 0 & 0 & t_{55}^2 + t_{56}^2 + t_{66}^2 \end{pmatrix}$$

The bias matrix can be used as a measure of the degree of the confounding among effects. For example, the coefficients corresponding to h_3 and h_4 (factor B) in the calculation of sum of squares of A are t_{23} and t_{24}; thus $t_{23}^2 + t_{24}^2$ (squaring is to avoid the sign) can be used as a confounding index between A and B.

1.14 Redundant Effects

If there are empty cells in the design, some effects in the model may not be estimable. MANOVA determines the redundant effects by orthonormalization of the design matrix and prints the information. Figure 1.14 indicates that the interaction effects in columns 10 and 12 in the design matrix are not estimable because of empty cells.

Figure 1.14

```
REDUNDANCIES IN DESIGN MATRIX

  COLUMN      EFFECT

    10      A BY B
    12       (SAME)
```

1.15 Solution Matrices

For any connected design, the hypotheses associated with the sequential sums of squares are weighted functions of the population cell means with weights depending on the cell frequencies (e.g. see Searle(1971), pp. 306-313). For designs with every cell filled, it can be shown that the hypotheses corresponding to the regression model sums of squares are the unweighted hypotheses about the cell means. With empty cells the hypotheses will depend on the pattern of the missingness. In such cases, one can request that the solution matrix, which contains the coefficients of the linear combinations of the cell means being tested, be printed by specifying