ASSUMPTION OF ADDITIVITY OF EFFECTS

A basic assumption of the experimental designs described in this book is that a score is the *sum* of the effects in the linear model. If the assumption of additivity of effects is not tenable, it may be possible to achieve additivity by a suitable transformation of the scores.

References that provide additional discussion of the assumptions in analysis of variance may be found in the papers by Eisenhart (1947) and Cochran (1947).

2.7 TRANSFORMATIONS

A transformation is any systematic alteration in a set of scores whereby certain characteristics of the set are changed and other characteristics remain unchanged. Three major reasons for using transformations in analysis of variance are

1. To achieve homogeneity of error variance.
2. To achieve normality of treatment-level distributions (or within-cell distributions).
3. To obtain additivity of treatment effects.

Because the F distribution is relatively unaffected by lack of normality and heterogeneity of variance, the first two reasons for performing a transformation are less compelling than the third. Obtaining additivity of effects is particularly important in designs such as a randomized block design in which a *residual* mean square (abbreviated MS_{res}) is used as an estimate of experimental error. For example, if treatment levels and blocks are not additive, the expected value of the residual mean square is

$$E(MS_{res}) = \sigma_\varepsilon^2 + \sigma_{\beta\pi}^2$$

instead of

$$E(MS_{res}) = \sigma_\varepsilon^2,$$

where $\sigma_{\beta\pi}^2$ refers to the interaction of treatment levels and blocks. Interaction in this context is said to be present when the dependent variable that is measured under the k treatment levels behaves differently for different blocks of subjects. The expected value of the treatment mean square for a fixed-effects model is

$$E(MS_B) = \sigma_\varepsilon^2 + n\sigma_\beta^2.$$

If the null hypothesis is true, then, according to Section 2.5, the numerator and denominator of the ratio