$$\hat{\sigma}_j^2 = \left[\sum_1^n X^2 - \frac{\left(\sum_1^n X\right)^2}{n}\right]\Big/(n-1),$$

$MS_{error} = \Sigma_{j=1}^k \hat{\sigma}_j^2/v$, and k = number of variances. For values of $v_j \geq 5$, B is approximately distributed as the χ^2 distribution, with $k - 1$ degrees of freedom. If $v_j < 5$, tables prepared by Merrington and Thompson (1946) may be used.

Two other tests are computationally simpler than Bartlett's test and provide an adequate test of the assumption of homogeneity of variance. The simpler of the two tests, which was proposed by Hartley (1940, 1950), uses the statistic F_{max},

$$F_{max} = \frac{\text{largest of } k \text{ variances}}{\text{smallest of } k \text{ variances}} = \frac{\hat{\sigma}_{j\,\text{largest}}^2}{\hat{\sigma}_{j\,\text{smallest}}^2},$$

with degrees of freedom equal to k and $n - 1$, where k is the number of variances and n is the number of observations within each treatment level. The distribution of F_{max} is given in Table D.10. The hypothesis of homogeneity of variance is rejected if F_{max} is greater than the tabled value for $F_{max,\alpha}$. If the n's for the treatment levels differ only slightly, the largest of the n's can be used for purposes of determining the degrees of freedom for this test. This procedure leads to a slight positive bias in the test, that is, in rejecting the hypothesis of homogeneity more frequently than it should be rejected.

The other relatively simple test of homogeneity of variance is that proposed by Cochran (1941). This test statistic is given by

$$C = \frac{\hat{\sigma}_{j\,\text{largest}}^2}{\sum_{j=1}^k \hat{\sigma}_j^2},$$

where $\hat{\sigma}_{j\,\text{largest}}^2$ is the largest of the k treatment variances and $\Sigma_{j=1}^k \hat{\sigma}_j^2$ is the sum of all of the variances. The degrees of freedom for this test are equal to k and $n - 1$ as defined for the F_{max} test. The sampling distribution of C is given in Table D.11.

Since the F distribution is so robust with respect to violation of the assumption of homogeneity of error variance, it is not customary to test this assumption routinely. Both the Hartley and the Cochran tests have adequate sensitivity for testing the assumption in situations where heterogeneity is suspected. If variances are heterogeneous, a transformation of scores as described in Section 2.7 may produce homogeneity.

It should be noted that all three tests described here are sensitive to departures from normality as well as heterogeneity of variances (Box and Anderson, 1955). For a description of a test that is relatively insensitive to departures from normality, see Odeh and Olds (1959).