

Studies by Pearson (1931) and Norton, as cited by Lindquist (1953), indicate that the  $F$  distribution is relatively unaffected by lack of symmetry of treatment populations. It is also relatively unaffected by kurtosis except in extreme cases of very leptokurtic or platykurtic populations. For the fixed-effects model, an experimenter need not be concerned if the  $k$  populations exhibit a moderate departure from the normal distribution provided that the  $k$  populations are homogeneous in form, for example, all treatment populations positively skewed and slightly leptokurtic. In general, unless the departure from normality is so extreme that it can be readily detected by visual inspection of the data, the departure will have little effect on the probability associated with the test of significance. It may be possible to transform nonnormally distributed scores so as to achieve normality, under conditions described in Section 2.7.

ASSUMPTION OF HOMOGENEITY OF POPULATION-ERROR VARIANCES

The  $F$  distribution is robust with respect to violation of the assumption of homogeneity of population-error variances provided that the number of observations in the samples is equal (Cochran, 1947; Norton as cited by Lindquist, 1953). However, for samples of unequal size, violation of the homogeneity assumption can have a marked effect on the test of significance. According to Box (1953, 1954a), the nature of the bias for this latter case may be positive or negative.

Several statistics are available for testing the homogeneity assumption that

$$H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2 = \sigma_e^2.$$

The alternative to the above null hypothesis is

$$H_1: \text{some } \sigma_j^2 \text{'s are unequal.}$$

A test statistic proposed by Bartlett (1937) is

$$B = \frac{2.30259}{C} \left[ v \log_{10} MS_{\text{error}} - \sum_{j=1}^k (v_j \log_{10} \hat{\sigma}_j^2) \right],$$

where

$$C = 1 + \frac{\sum_{j=1}^k \frac{1}{v_j} - \frac{1}{v}}{3(k-1)},$$

$v_j$  = degrees of freedom for  $\hat{\sigma}_j^2$ ,  $v$  = degrees of freedom for  $MS_{\text{error}}$  equal to  $\sum_{j=1}^k v_j$ ,  $\hat{\sigma}_j^2$  = unbiased estimate of population variance for the  $j$ th population given by