

distribution of  $F$  and those associated with the mathematical model for a particular design—the  $F$  ratio can be used to test the hypothesis that all treatment population means are equal.

## 2.6 EFFECTS OF FAILURE TO MEET ASSUMPTIONS IN ANALYSIS OF VARIANCE

The emphasis in the previous sections of this chapter has been on the assumptions necessary for the mathematical justification of hypothesis-testing procedures using the  $F$  distribution. What are the consequences of failure to meet these assumptions? Cochran and Cox (1957, 91) stated that failure to meet the assumptions affects both the significance level of a test and the sensitivity of a test. For example, a test performed at the .05 level may actually be made at the .04 or .07 level. Also, a loss in sensitivity results when the assumptions are not fulfilled because it is often possible to construct a more powerful test than that using the  $F$  ratio if the correct model can be specified. Fortunately, the  $F$  distribution is very robust with respect to violation of many of the assumptions associated with its mathematical derivation. The effects of failure to meet certain assumptions associated with the  $F$  distribution and the mathematical model for a design are discussed in the following paragraphs. Cochran (1947) has pointed out that it is impossible to be certain that all required assumptions are exactly satisfied by a set of data. Thus analysis of variance must be regarded as approximate rather than exact. However, it is generally possible, by a careful examination of the data, to detect cases in which a standard analysis will lead to gross errors in interpreting the outcome of an experiment.

### ASSUMPTION OF NORMALLY DISTRIBUTED POPULATION

One of the requirements in order for an  $F$  ratio to be distributed as the  $F$  distribution is that the numerator and denominator of the ratio are independent. If scores are randomly sampled from a normal population, this requirement is satisfied.

An assumption of both the fixed-effects and random-effects models is that the errors  $\varepsilon_{ij}$  are normally distributed for each treatment population. Because the only source of variation within a treatment population are the errors, the assumption of normally distributed  $\varepsilon_{ij}$ 's is equivalent to the assumption of normally distributed scores.

A population of scores can depart from the normal distribution in terms of either skewness or kurtosis, or in both skewness and kurtosis.