sample mean had been 102.5 instead of 102, the decision would have been to reject H_0. This can be demonstrated as follows:

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{102.5 - 100.0}{1.5} = 1.67.$$

According to the normal distribution table in Appendix D.3, the probability of obtaining a sample mean of 102.5 if the true mean is really 100 is less than .05. Thus, either a rare and improbable event has occurred or the true parameter is not 100. This example was fabricated to illustrate the steps involved in testing a statistical hypothesis. In real life most hypotheses are concerned not with a single population but with differences among two or more populations. The steps that have been described in connection with a single population are also applicable to tests involving two or more populations. Procedures and assumptions associated with testing statistical hypotheses with respect to two or more populations are described in Chapters 2 and 3. An excellent survey of hypothesis testing and statistical inference is given by Clark (1963).

TYPE I AND II ERRORS

In carrying out the decision process outlined above, the experimenter may make a correct decision or he may commit an error. If he decides to reject H_0 when the population mean is really equal to 100, he has committed a type I error. On the other hand, if he decides not to reject H_0 when the population mean is really equal to, say, 103, he has committed a type II error. In summary, the two possible errors an experimenter may make are

Type I error. Reject H_0 (tested hypothesis) when it is true. The probability α is the risk of making a type I error.

Type II error. Fail to reject H_0 when it is false. The risk of making a type II error is designated as β.

The regions corresponding to the probability of making a type I error (α) and a type II error (β) are shown in Figure 1.5-3.

It is apparent from Figure 1.5-3 that the probability of making a type I error is determined by an experimenter when he specifies α. This probability can be made as small as an experimenter wishes. It should be noted from the figure that as the area corresponding to α is made smaller, the area designated as β becomes larger. Thus the two types of errors are interrelated. The probability of committing a type II error is determined by α, magnitude of difference between the true parameter and parameter under H_0, size of population error variance, and size of sample (n). If, in the hypothesis-testing example described previously, the statistic is equal to 102, a decision is made not to reject H_0. This decision may be correct or incorrect, depending on the value of the parameter. If