of a z ratio is a constant for any sample of size n because the population parameter σ is a constant. By comparison, the denominator of a t ratio for any sample of size n is a random variable because of sampling variation in estimating the parameter σ. The numerators of both test statistics are subject to sampling variation and hence are random variables.

In practice, the population variance required to compute z is rarely ever known. The ratio

$$\frac{\bar{X} - \mu_0}{\hat{\sigma}/\sqrt{n}}$$

can be treated as a z variable provided that the sample size is large, say around 100, and that the population has a normal distribution. An examination of Appendix Tables D.3 and D.4 reveals that the probabilities associated with values of z and t are quite similar even for samples as small as 30. As the sample size is reduced below 30, the correspondence becomes poorer. Thus, for small samples, the t distribution should be employed if a *sample* standard deviation is used to estimate σ. The t distribution and other useful sampling distributions are discussed in Section 2.1.

EXAMPLE ILLUSTRATING STEPS IN HYPOTHESIS TESTING

An example may help to clarify the concepts and conventions involved in hypothesis testing. Assume that we wish to test the hypothesis that the average performance of some population on a psychological test is greater than 100. An arithmetic mean is chosen as the appropriate measure of central tendency. Two *test statistics* can be suggested for this experiment:

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \quad \text{and} \quad t = \frac{\bar{X} - \mu_0}{\hat{\sigma}/\sqrt{n}}.$$

Let us assume in our example that the population is normally distributed and that σ is known to equal 15. Under these conditions, the appropriate test statistic is z. The statistical hypotheses can be stated as follows:

$$H_0: \mu \leq 100$$

$$H_1: \mu > 100.$$

We will reject the null hypothesis in favor of the alternative hypothesis only if an observed sample mean is so much larger than 100 that it has a probability of .05 or less of occurring if the population mean really is equal to 100. As written, the null hypothesis is inexact because it states a whole region of possible values for the population mean. However, one exact value is specified, $\mu = 100$. Actually, the hypothesis tested is $\mu = 100$