If we let τ_{ij} stand for all treatment effects, the error effect can be written

$$\hat{\varepsilon}_{m(ij)} = X_{ijm} - \hat{\tau}_{ij} - \hat{\mu}.$$

In this form the similarity between the error effect for this design and the error effect for a completely randomized design is apparent. This latter error effect was given earlier as

$$\hat{\varepsilon}_{ij} = X_{ij} - \hat{\beta}_j - \hat{\mu}.$$

The similarity between the models for a completely randomized design and a completely randomized factorial design is not surprising in view of the fact that the former design is the *building block* for the latter design.

The error effect for a randomized block factorial design is

$$\hat{\varepsilon}_{ijm} = X_{ijm} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\pi}_m - \widehat{\alpha\beta}_{ij} - \hat{\mu}.$$

TABLE 1.4-6 Completely Randomized Factorial Design

	Radiation Levels			
Temperature Levels	b_1	b_2	b_3	*A* treatment means
a_1	X_{111} X_{112} X_{113}	X_{121} X_{122} X_{123}	X_{131} X_{132} X_{133}	$\bar{X}_{1\cdot\cdot}$
a_2	X_{211} X_{212} X_{213}	X_{221} X_{222} X_{223}	X_{231} X_{232} X_{233}	$\bar{X}_{2\cdot\cdot}$
B treatment means =	$\bar{X}_{\cdot 1\cdot}$	$\bar{X}_{\cdot 2\cdot}$	$\bar{X}_{\cdot 3\cdot}$	Grand mean = $\bar{X}_{\cdots}$

TABLE 1.4-7 Randomized Block Factorial Design

Temperature Levels	a_1	a_1	a_1	a_2	a_2	a_2	
Radiation Levels	b_1	b_2	b_3	b_1	b_2	b_3	Block mean
Block (litter) p_1	X_{111}	X_{121}	X_{131}	X_{211}	X_{221}	X_{231}	$\bar{X}_{\cdot\cdot 1}$
Block (litter) p_2	X_{112}	X_{122}	X_{132}	X_{212}	X_{222}	X_{232}	$\bar{X}_{\cdot\cdot 2}$
Block (litter) p_3	X_{113}	X_{123}	X_{133}	X_{213}	X_{223}	X_{233}	$\bar{X}_{\cdot\cdot 3}$
Column means	$\bar{X}_{11\cdot}$	$\bar{X}_{12\cdot}$	$\bar{X}_{13\cdot}$	$\bar{X}_{21\cdot}$	$\bar{X}_{22\cdot}$	$\bar{X}_{23\cdot}$	Grand mean = $\bar{X}_{\cdots}$

A_1 treatment mean = $(X_{111} + X_{112} + X_{113} + X_{121} + \cdots + X_{133})/9 = \bar{X}_{1\cdot\cdot}$

A_2 treatment mean = $(X_{211} + X_{212} + X_{213} + X_{221} + \cdots + X_{233})/9 = \bar{X}_{2\cdot\cdot}$

B_1 treatment mean = $(X_{111} + X_{112} + X_{113} + X_{211} + X_{212} + X_{213})/6 = \bar{X}_{\cdot 1\cdot}$

B_2 treatment mean = $(X_{121} + X_{122} + X_{123} + X_{221} + X_{222} + X_{223})/6 = \bar{X}_{\cdot 2\cdot}$

B_3 treatment mean = $(X_{131} + X_{132} + X_{133} + X_{231} + X_{232} + X_{233})/6 = \bar{X}_{\cdot 3\cdot}$