subjects are assigned to the treatment levels so that each possible pair of treatment levels occurs together within some block an equal number of times. A design having these characteristics is called a *balanced* incomplete block design. *Partially balanced* designs are those in which some pair of treatment levels occur together within the blocks more often than do other pairs.

FACTORIAL EXPERIMENT

A factorial experiment permits an investigator to evaluate the combined effects of two or more treatments in a single experiment. This is accomplished by combining building block designs so that one level from each of two or more treatments is presented simultaneously. The most commonly used building block designs are the completely randomized design and the randomized block design.

In the microwave radiation example, an investigator can, by using a factorial experiment, evaluate the effects of radiation and also the effects of a second treatment, such as room temperature. Assume that there are two levels of ambient room temperature, $a_1 = 80°$ and $a_2 = 65°$, and three levels of radiation, $b_1 = 0$, $b_2 = 20{,}000$, and $b_3 = 40{,}000$ microwatts. Tables 1.4-6 and 1.4-7 illustrate the use of two frequently used building block designs in a factorial experiment. In Table 1.4-6 the three subscripts designate a particular temperature level, radiation level, and subject, in that order. The three subscripts in Table 1.4-7 designate a particular temperature level, radiation level, and block, in that order.

In the completely randomized factorial design of Table 1.4-6 it is assumed that 18 albino rats are randomly assigned to the six treatment level combinations. In the randomized block factorial design example shown in Table 1.4-7, the treatment level combinations are randomly assigned within each block of litter mates. The models for the completely randomized and randomized block factorial designs are, respectively,

$$X_{ijm} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{m(ij)}$$

$$X_{ijm} = \mu + \alpha_i + \beta_j + \pi_m + \alpha\beta_{ij} + \varepsilon_{ijm}.$$

The effect of temperature is designated by α_i, radiation by β_j, interaction of temperature and radiation by $\alpha\beta_{ij}$, experimental error by ε, and litter by π_m. Both designs permit an investigator to determine if radiation dosage has the same effect on food consumption at 80° ambient room temperature as it has at 65° ambient room temperature. It is conceivable that radiation might be more detrimental at a high ambient room temperature than at a low ambient room temperature. If such a result is found, it is called an *interaction effect*.

The error effect for a completely randomized factorial design, using the scheme described previously, can be written

$$\hat{\varepsilon}_{m(ij)} = X_{ijm} - \hat{\alpha}_i - \hat{\beta}_j - \widehat{\alpha\beta}_{ij} - \hat{\mu}.$$