The three subscripts designate a particular block, weight category, and treatment level, in that order. The three treatment levels c_k are randomly assigned to the nine cells with the restriction that each treatment level must occur in any row and any column only once. In order to achieve this balance, a Latin square design must have the same number of rows, columns, and treatment levels. Consequently, only 9 animals can be used in the design shown in Table 1.4-4 instead of the 15 animals used in the two designs described previously.

The linear model for this design is

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}.$$

An individual score is equal to the grand mean μ, plus a block effect α_i, plus a column effect β_j, plus a treatment effect γ_k, plus an error effect ε_{ijk}. If the block and column effects, α_i and β_j, in a Latin square design are appreciably greater than zero, the design may be more powerful than either a completely randomized or a randomized block design. This is apparent if the error effect is examined by means of the procedure used for the two designs described previously. The error effect for a Latin square design is equal to

$$\hat{\varepsilon}_{ijk} = X_{ijk} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k - \hat{\mu}.$$

INCOMPLETE BLOCK DESIGN

An incomplete block design is particularly applicable to research situations in which the number of subjects available for each block is less than the number of treatment levels. If, for example, only 2 albino rats from each litter are available, and the experimenter wants to use three treatment levels, an incomplete block design is required. This design is shown in Table 1.4-5.

TABLE 1.4-5 Incomplete Block Design

	Treatment Levels			
	b_1	b_2	b_3	Block means
Block (litter)	X_{11}		X_{13}	$\bar{X}_{1\cdot}$
Block (litter)		X_{22}	X_{23}	$\bar{X}_{2\cdot}$
Block (litter)	X_{31}	X_{32}		$\bar{X}_{3\cdot}$
Treatment means =	$\bar{X}_{\cdot 1}$	$\bar{X}_{\cdot 2}$	$\bar{X}_{\cdot 3}$	Grand mean = $\bar{X}_{\cdot\cdot}$

The linear model for this design is

$$X_{ij} = \mu + \beta_j + \pi_i + \varepsilon_{ij}.$$

It should be noted that each block contains the same number of subjects, each treatment level occurs the same number of times, and