in the previous example were taken from five different litters. Rats from the same litter can be expected to be more homogeneous with respect to genetic characteristics than rats from different litters. In Table 1.4-3 the 3 rats in each row that comprise a block are from the same litter. Differences among the litters can be regarded as a nuisance variable that is experimentally isolated through the use of a randomized block design. The subscripts of X_{ij} designate a particular litter and treatment level, in that order. Differences among the column means reflect treatment effects, whereas differences among the row means reflect litter effects.

TABLE 1.4-3 Randomized Block Design

	Treatment Levels			
	b_1	b_2	b_3	Block means
Block (litter) p_1	X_{11}	X_{12}	X_{13}	$\bar{X}_{1\cdot}$
Block (litter) p_2	X_{21}	X_{22}	X_{23}	$\bar{X}_{2\cdot}$
Block (litter) p_3	X_{31}	X_{32}	X_{33}	$\bar{X}_{3\cdot}$
Block (litter) p_4	X_{41}	X_{42}	X_{43}	$\bar{X}_{4\cdot}$
Block (litter) p_5	X_{51}	X_{52}	X_{53}	$\bar{X}_{5\cdot}$
Treatment means =	$\bar{X}_{\cdot 1}$	$\bar{X}_{\cdot 2}$	$\bar{X}_{\cdot 3}$	Grand mean = $\bar{X}_{\cdot\cdot}$

Assignment of the three treatment levels to the rats is randomized independently for each row. The linear model for this design is

$$X_{ij} = \mu + \beta_j + \pi_i + \varepsilon_{ij}.$$

Unbiased estimates of the parameters are given by the statistics

$$\hat{\mu} = \bar{X}_{\cdot\cdot} \rightarrow \mu$$
$$\hat{\beta}_j = (\bar{X}_{\cdot j} - \bar{X}_{\cdot\cdot}) \rightarrow \beta_j$$
$$\hat{\pi}_i = (\bar{X}_{i\cdot} - \bar{X}_{\cdot\cdot}) \rightarrow \pi_i$$
$$\hat{\varepsilon}_{ij} = (X_{ij} - \bar{X}_{\cdot j} - \bar{X}_{i\cdot} + \bar{X}_{\cdot\cdot}) \rightarrow \varepsilon_{ij}.$$

The term π_i represents an effect attributable to the ith block of 3 rats. It can be shown, by regrouping terms in the linear model and substituting statistics for parameters, that the error effect in a randomized block design is equal to

$$\hat{\varepsilon}_{ij} = X_{ij} - \hat{\beta}_j - \hat{\pi}_i - \hat{\mu}.$$

The error effect for a completely randomized design was given earlier as

$$\hat{\varepsilon}_{ij} = X_{ij} - \hat{\beta}_j - \hat{\mu}.$$

Thus the error effect $\hat{\varepsilon}_{ij}$ for a randomized block design is equal to the completely randomized design error effect minus a block effect $\hat{\pi}_i$. It is apparent from this that the error effect for a randomized block design will be smaller than the error effect for a completely randomized design if the block effect $\hat{\pi}_i$ is appreciably greater than zero.