(1) $$X_{ij} = \mu + \beta_j + \varepsilon_{ij}.$$

According to this model, an individual score is equal to the population mean μ, plus a treatment effect β_j, plus an error effect ε_{ij}, which is unique for each individual subject. In a particular experiment, the parameters μ, β_j, and ε_{ij} are unknown, but sample estimates of these parameters are given by $\hat{\mu}$, $\hat{\beta}_j$, and $\hat{\varepsilon}_{ij}$, respectively. It can be shown by maximum-likelihood methods that unbiased estimates of the required parameters are provided by the statistics

$$\hat{\mu} = \bar{X}.. \quad \rightarrow \mu$$

$$\hat{\beta}_j = (\bar{X}._j - \bar{X}..) \rightarrow \beta_j$$

$$\hat{\varepsilon}_{ij} = (X_{ij} - \bar{X}._j) \rightarrow \varepsilon_{ij}.$$

The symbol → indicates that the term on the left is an estimator of the term on the right. According to the maximum-likelihood method, the best estimate is the one that gives the highest probability of obtaining the observed data. It should be noted that a maximum-likelihood estimator is not necessarily unbiased, although the center of its distribution is generally close to the value of the parameter estimated. Assumptions associated with the mathematical model for a completely randomized design are discussed in Chapter 2 and explicitly stated in connection with the description of each design in subsequent chapters.

The meaning of the term *error effect* is somewhat elusive. An intuitive understanding of this term can be obtained by an examination of Table 1.4-2 and the linear model for the design. It is obvious that the scores for all 5 rats exposed to treatment level b_1 in this table will probably not be identical. Variation among the five scores can be attributed to a variety of sources—experiences of the rats prior to participation in the experiment, unintended variation in administration of the treatment level, lack of reliability in measuring the effect of the treatment level, etc. An error effect is an estimate of all effects *not* attributable to a particular treatment level. This can be seen from the linear model if the terms in equation (1) are rearranged and statistics are substituted for the parameters. The equation can be written

$$\hat{\varepsilon}_{ij} = X_{ij} - \hat{\beta}_j - \hat{\mu}.$$

Thus the error effect is that portion of a score remaining after the treatment effect and grand mean are subtracted from it. An experimenter attempts, by using an appropriate design and experimental controls, to minimize the size of the error effect. Designs described in subsequent paragraphs permit an experimenter to accomplish this by isolating additional sources of variation that affect individual scores.

RANDOMIZED BLOCK DESIGN

A randomized block design is based on the principle of assigning subjects to blocks so that the subjects within each block are more homogeneous than subjects in different blocks. Assume that the 15 albino rats